BRIEF COMMUNICATION

A SURFACE RENEWAL MODEL FOR INTERFACIAL HEAT AND MASS TRANSFER IN TRANSIENT TWO-PHASE FLOW

S. BANERJEE

Department of Engineering Physics, McMaster University, Hamilton, Ontario, L8S 4M1, Canada

INTRODUCTION

To analyse non-equilibrium two-phase flow, it is necessary to use not only the conservation equations for each phase, but also constitutive relationships for the transfer of mass, energy and momentum at the interface and wall. One of the central problems at present is to obtain the form of the constitutive relationships for the interfacial transfer rates. Progress in this direction has been summarized by Bouré (1975) using the consistency requirements between various one-dimensional two-phase flow models. He has shown that, in general, the constitutive relationships must consist of both algebraic and derivative terms of the dependent variables.

In the present paper, a model for interfacial energy and mass flux (per unit interfacial area) is proposed based on surface renewal theory, which also leads to algebraic and derivative terms for transient processes. The model requires only one free parameter, the surface renewal rate, for interfacial mass and energy flux.

Surface renewal (or penetration) theory has been used extensively in the chemical engineering literature to derive expressions for interfacial transfer rates in steady state operations (Bird *et al.* 1960). Surface renewal theory has been found to correctly predict that the interfacial flux is proportional to the square root of the diffusivity, whereas "film" theory incorrectly predicts that the interfacial flux is directly proportional to the diffusivity. Banerjee *et al.* (1968), Banerjee (1971), and Theofanous and co-workers (1975, 1976), amongst others, have also proposed models relating the surface renewal rate to parameters such as energy dissipation within a phase. Such models have been found to predict a wide range of mass transfer data in steady state operations such as gas absorption.

No work appears to have been published on the application of surface renewal theory to transient processes. As shown in this paper, such an application leads to somewhat modified expressions for the interfacial fluxes compared to the steady state expressions.

DERIVATION OF CONSTITUTIVE RELATIONSHIPS

Consider heat transfer from the interface into the fluid phase, k. Let the interfacial temperature be $T_{ik}(t)$ which is the saturation temperature at the interface pressure, $p_{ik}(t)$†. (The subscript i denotes conditions at the interface and the subscript k denotes phase k). In essence, surface renewal theory considers that turbulent eddies renew the interface by bringing fluid at the bulk fluid temperature T_k to the interface at average intervals of τ s (or at a surface renewal rate $s = 1/\tau\ s^{-1}$). The surface renewal rate depends on the turbulence structure near the interface. In between renewals, heat (or mass) is considered to transfer by molecular diffusion. Thus, the heat transfer problem reduces to finding the conduction heat transfer during the average interval between renewals. This interval is usually a very short period characteristic of the time scale of eddies at high wave numbers.

The conduction problem is solved by considering a system of Cartesian coordinates established at the interface and convecting with the interface velocity. The equations to be

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†There are arguments to suggest that p_{ik} should always be taken as the interfacial pressure on the vapour side in vapour–liquid flows.

solved are (assuming the length scale of the eddies are small compared to interface radius of curvature)

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2},$$ [1]

where x is the direction normal to the interface. The boundary conditions are

$$T = T_{ik} \text{ at } x = 0,$$

$$T = T_k \text{ at } t = 0, x \rightarrow \infty,$$

and

K = thermal diffusivity = $\hat{k}/\rho C_p$, $\hat{k}$ is the thermal conductivity, ρ the density and C_p the specific heat.

If the time interval between renewals is small, then $0 \leq t \leq \tau$ and t is small. Therefore,

$$T_{ik}(t) = T_{ik}(0) + \frac{dT_{ik}}{dt}\bigg|_{t=0} t + 0(t^2) \ldots$$ [2]

where t is measured from the last surface renewal. For terms of the order t^2 to be negligible, the maximum value of $t(=\tau)$ must be small in relation to the rate of change of interfacial temperature. The total derivative is taken with regard to the interfacial velocity.

$$T(x,t) - T_k = (T_{ik} - T_k) \, \text{erfc} \left(\frac{x}{2\sqrt{(\hat{k}t)}}\right) + \frac{dt_{ik}}{dt} 4ti^2 \, \text{erfc} \left(\frac{x}{2\sqrt{(\hat{k}t)}}\right).$$ [3]

The instantaneous heat flux into phase k may be derived by taking the derivative of [3] at $x = 0$,

$$-\hat{k}\frac{dT}{dx}\bigg|_{x=0} = \rho C_p \sqrt{\left(\frac{K}{\pi t}\right)} (T_{ik} - T_k) + \frac{dT_{ik}}{dt} \frac{\rho C_p \sqrt{(Kt)}}{\Gamma(3/2)},$$ [4]

where we have used

$$i\text{erfc}(0) = \frac{1}{2\Gamma(3/2)}.$$

The average heat flux during the period τ is (noting $\Gamma(3/2) = \sqrt{(\pi/2)}$),

$$-\hat{k}\frac{dT}{dx}\bigg|_{x=0} = 2\rho C_p \sqrt{\left(\frac{K}{\pi\tau}\right)} (T_{ik} - T_k) + \frac{dT_{ik}}{dt} \frac{4}{3}\rho C_p \sqrt{\left(\frac{K\tau}{\pi}\right)}.$$ [5]

The interfacial heat transfer into phase k per unit volume, q_{ik}, can be derived from [5] by using the interfacial area per unit volume, a_i, as

$$q_{ik} = a_i \left[2\rho C_p \sqrt{\left(\frac{K}{\pi\tau}\right)} (T_{ik} - T_k) + \frac{4}{3\sqrt{\pi}} \rho C_p \sqrt{(K\tau)} \left(\frac{dT_{ik}}{dp_{ik}}\right)_{\text{sat}} \frac{dp_{ik}}{dt} \right].$$ [6]

where the total derivative is taken with respect to the interfacial velocity. Note that the expression for interfacial heat transfer is of the form

$$q_{ik} = H_k a_i (T_{ik} - T_k) + H_k^1 a_i \frac{dp_{ik}}{dt},$$ [7]

where

$$H_k = 2\rho C_p \sqrt{\left(\frac{K}{\pi\tau}\right)},$$

$$H_k^1 = \frac{4}{3\sqrt{\pi}} \rho C_p \sqrt{(K\tau)} \left(\frac{dT_{ik}}{dp_{ik}}\right)_{sat}.$$

[8]

The term $(dT_{ik}/dp_{ik})_{sat}$ may be further simplified by the Clapeyron equation. The first term in [7] is the usual heat-transfer coefficient obtained by applying surface renewal theory to steady state operations. The second term, however, arises due to the transient nature of the process and it is clear that the expression for interfacial heat transfer will contain a derivative term with respect to the interfacial pressure. This term will be especially important during rapid depressurization or pressurization.

A second point of interest is that the theory leads to only one free parameter τ in the expressions for H_k and H_k^1. This parameter has some physical significance in that it is related to turbulence time scales. The mass transfer out of phase k may be written as

$$\dot{m}_k = \rho_k \mathbf{n}_k \cdot (\mathbf{v}_k - \mathbf{v}_i),$$

where $\mathbf{n}_k$ is the outward drawn normal from phase k, $\mathbf{v}_k$ is the velocity of phase k near the interface and $\mathbf{v}_i$ is the interface velocity. Also

$$\dot{m}_G = -\dot{m}_L = \frac{q_{iL} + q_{iG}}{a_i(h_{iG} - h_{iL})}$$

[9]

where h is the specific enthalpy. Thus, it is clear that the constitutive relationships for interfacial mass transfer in transient vaporization or condensation processes must also contain the pressure derivative terms present in the expressions for interfacial heat transfer. The mass transfer rates are also related to the surface renewal parameters through[9] and [6].

For steam–water flows under most practical conditions, the interfacial heat transfer in the gas phase is usually much smaller (less than 5%) than in the liquid phase. Thus a simplified expression for mass transfer may be obtained by neglecting q_{iG} (where subscript G and L denote the gas and liquid phases respectively) as

$$\dot{m}_G = -\dot{m}_L = \frac{q_{iL}}{a_i(h_{iG} - h_{iL})}.$$

[10]

(Note that $\dot{m}_G$ is mass transfer *out* of the vapour phase, whereas q_{iL} is interfacial heat transfer *into* the liquid phase). Thus, for many practical applications, it may be sufficient to estimate the liquid surface renewal rate (and interfacial area) in order to determine interfacial mass and heat transfer rates. In these cases, it may also be possible to identify the interface velocity with the interfacial velocity of the liquid phase when calculating the total derivative of the interfacial pressure in[6].

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